Day 3: Review of Motion Along a Line

My students study motion along a line early in the year, so this assignment is a review for them. I like to spend a day on motion along a line as a segue into motion along a curve. For an excellent introduction to motion along a line, see the Curriculum Module on motion by Dixie Ross at AP Central[®]. (http://apcentral.collegeboard.com/apc/public/ repository/AP_CurricModCalculusMotion.pdf)

Day 3 Homework

The following problems are from old AP Exams and the sample multiple-choice problems in the Course Description, available at AP Central (http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2118.html).

Multiple-Choice Items:

1. 2003 AP Calculus AB Exam, Item 25 (no calculator):

A particle moves along the *x*-axis so that at time $t \ge 0$ its position is given by

 $x(t) = 2t^{3} - 21t^{2} + 72t - 53$. At what time *t* is the particle at rest? (A) t = 1 only (B) t = 3 only (C) $t = \frac{7}{2}$ only (D) t = 3 and $t = \frac{7}{2}$ (E) t = 3 and t = 4

- 2. 1998 AP Calculus AB Exam, Item 24 (no calculator): The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is
 - (A) 9
 - (B) 12
 - (C) 14
 - (D) 21
 - (E) 40

3. AP Calculus AB, sample multiple-choice Item 9 (no calculator): The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \ge 0$.

For what values of *t* is the speed of the particle increasing?

- (A) 3 < t < 4 only
 (B) t > 4 only
 (C) t > 5 only
 (D) 0 < t < 3 and t > 5
- (E) 3 < t < 4 and t > 5
- 4. 2003 AP Calculus AB Exam, Item 76 (calculator): A particle moves along the *x*-axis so that at any time *t* ≥ 0, its velocity is given by v(*t*) = 3 + 4.1cos(0.9*t*). What is the acceleration of the particle at time *t* = 4?
 (A) -2.016
 (B) -0.677
 (C) 1.633
 (D) 1.814
 (E) 2.97
- 5. 2003 AP Calculus AB Exam, Item 91 (calculator): A particle moves along the *x*-axis so that at any time t > 0, its acceleration is given by $a(t) = \ln(1+2^t)$. If the velocity of the particle is 2 at time t = 1, then the velocity of the particle at time t = 2 is

(A) 0.462

- (B) 1.609
- (C) 2.555
- (D) 2.886
- (E) 3.346

- 6. AP Calculus AB, sample multiple-choice Item 19 (calculator): Two particles start at the origin and move along the *x*-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of *t* do the particles have the same velocity?
 - (A) None
 - (B) One
 - (C) Two
 - (D) Three
 - (E) Four
- 7. AP Calculus AB, sample multiple-choice Item 15 (calculator):

A particle travels along a straight line with a velocity of $v(t) = 3e^{\left(-\frac{t}{2}\right)} \sin(2t)$ meters per second. What is the total distance traveled by the particle during the time interval $0 \le t \le 2$ seconds?

- (A) 0.835
- (B) 1.850
- (C) 2.055
- (D) 2.261
- (E) 7.025

Free-Response Questions:

- **8.** 2004 AP Calculus AB Exam, FRQ 3 (calculator): A particle moves along the *y*-axis so that its velocity at time $t \ge 0$ is given by
 - $v(t) = 1 \tan^{-1}(e^t)$. At time t = 0, the particle is at y = -1. (Note: $\tan^{-1} x = \arctan x$.)
 - (a) Find the acceleration of the particle at time t = 2.
 - (b) Is the speed of the particle increasing or decreasing at time t = 2? Give a reason for your answer.
 - (c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.
 - (d) Find the position of the particle at time t = 2. Is the particle moving toward the origin or away from the origin at time t = 2? Justify your answer.

9. 2006 AP Calculus AB/BC Exams, Item 4 (no calculator):

t (seconds)	0	10	20	30	40	50	60	70	80
v(t) (feet per second)	5	14	22	29	35	40	44	47	49

Rocket *A* has positive velocity v(t) after being launched upward from an initial height of 0 feet at time t = 0 seconds. The velocity of the rocket is recorded for selected values of *t* over the interval $0 \le t \le 80$ seconds, as shown in the table above.

- (a) Find the average acceleration of rocket *A* over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.
- (**b**) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- (c) Rocket *B* is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per

second. At time t = 0 seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time t = 80 seconds? Explain your answer.